**CZECH TECHNICAL UNIVERSITY IN PRAGUE**
**Faculty of Civil Engineering**
K123 – Department of Materials Engineering and Chemistry

# BM01 – **BUILDING MATERIALS**

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Lecture 3

# Mechanical properties

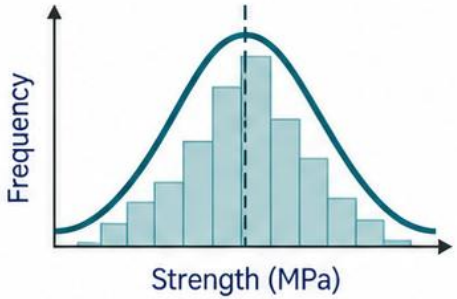
## 1 Loading and stress

- type of loading
- force and stress
- stress units

## 2 Strength

- resistance to failure
- strength types
- test specimens

## 3 Statistical evaluation



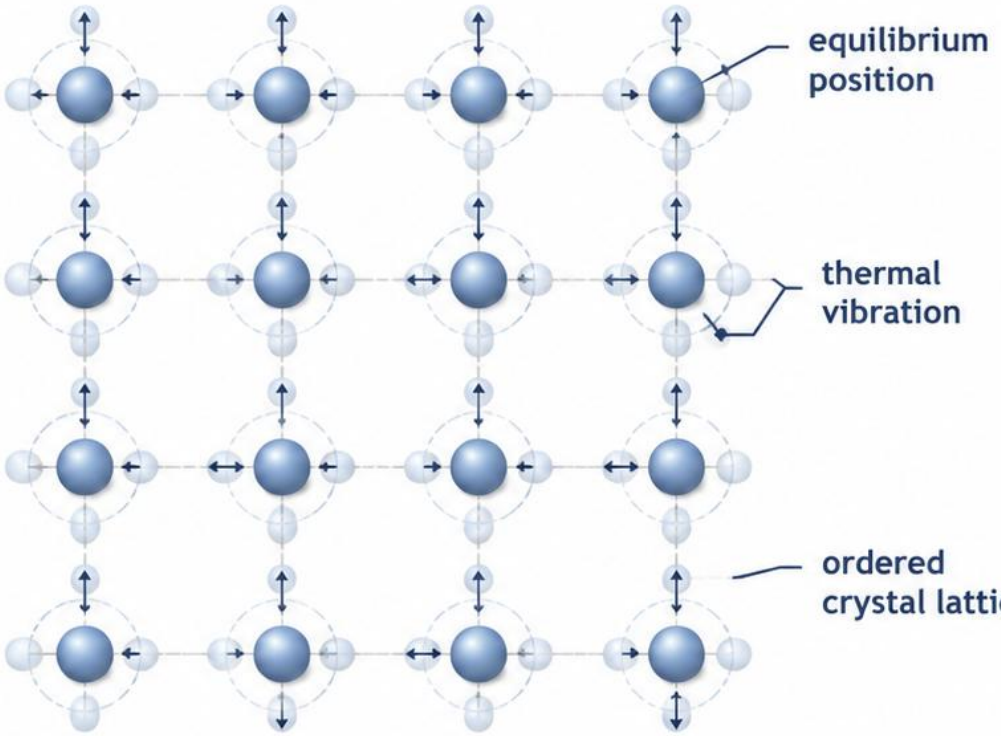

- mean and variability
- normal distribution
- guaranteed strength

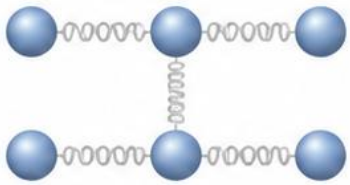
## 4 Testing modes

# Solid materials

**Key idea**

- structural rigidity
- resistance to changes in shape and volume
- atoms are tightly bound and only vibrate around fixed equilibrium positions



Atoms are bound to their neighbours.

Strong interatomic forces keep the lattice intact.

# Mechanical properties

- describe how a material responds to applied loads
- include deformation behaviour and resistance to failure

**load → response**

**UNLOADED**
initial state

**DEFORMATION**
change in shape or size

**FAILURE**
loss of load-bearing capacity

## Examples of mechanical properties

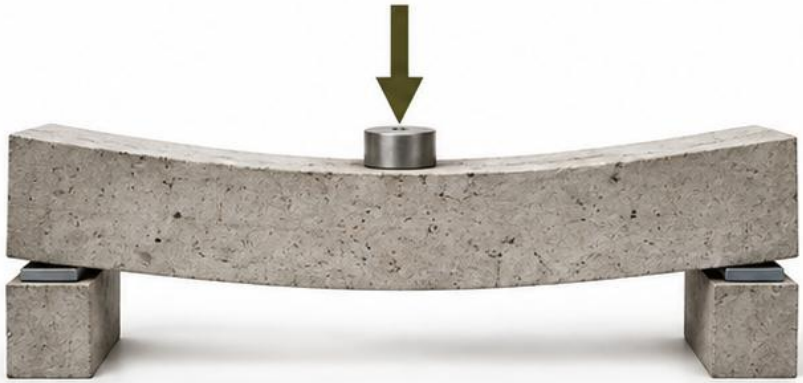
- **stiffness**
- **strength**
- **hardness**
- **toughness**
- **ductility**

# Mechanical properties

## Deformation properties

**material response to loading**

change in shape or dimensions under load

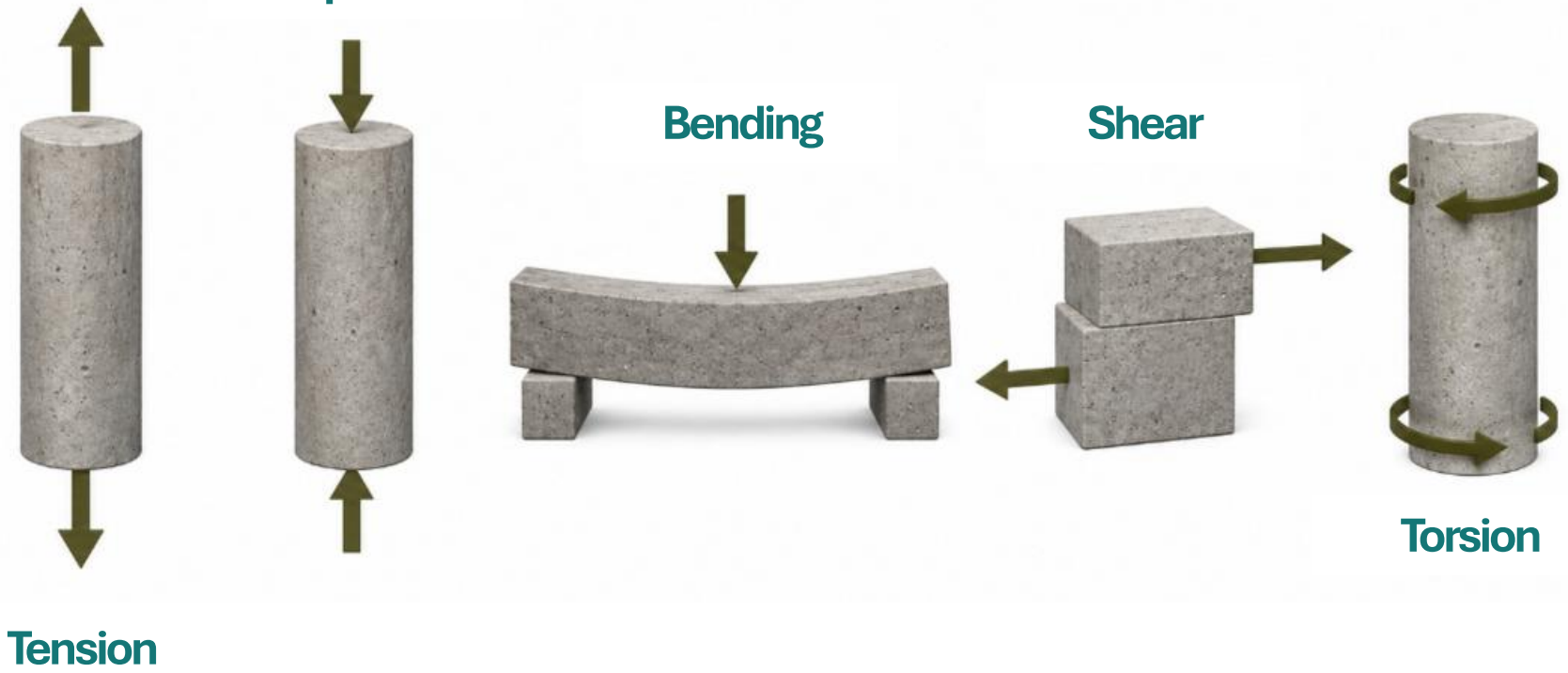


## Strength properties

**material resistance to failure**

maximum load the material can withstand before failure

# Basic loading modes



# Force and stress

**Force *F*** is an external load acting on a body

**Stress *σ*** describes how this load is transmitted within the material: as an internal force per unit area

## Force *F* [N]

For uniform axial loading

$$\sigma = F / A$$

$1\ Pa = 1\ N/m^2$

## Stress *σ* [Pa]

# Compressive stress

Compressive stress develops when a force presses a material over a contact area.

Average compressive stress over area A

$$\sigma = F / A$$

F – compressive force [N]

A – loaded contact area [m²]

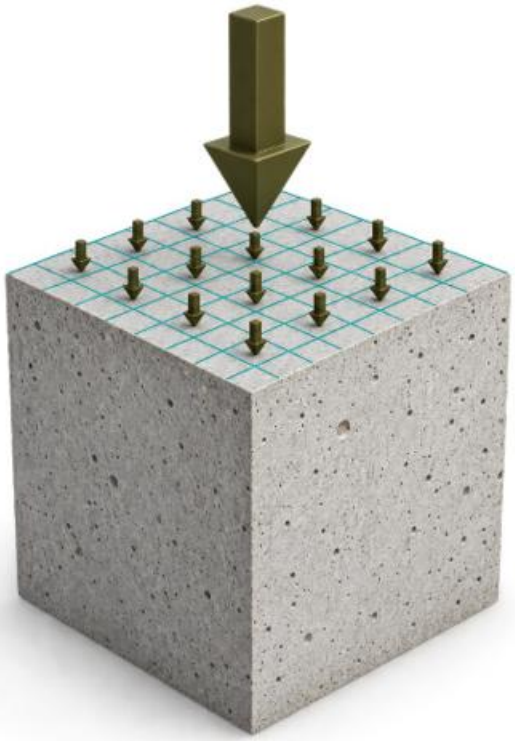
**Example: heel contact area**

$$A = 5\ mm \times 5\ mm = 25\ mm^2$$

$$= 2.5 \times 10^{-5}\ m^2$$

**Smaller contact area → greater stress**

# Units of stress



force distributed over a loaded area

Stress has dimensions of force per unit area. The unit follows from the chosen force and area units.

KEY SI EQUIVALENCE

**1 MPa = 1 N/mm²**

same numerical value • 1 MPa = $10^6$ Pa

OTHER UNIT RELATIONS

| | |
|---|---|
| Pa | 1 N/m² |
| psi | 1 lbf/in² |
| ksi | 1000 psi |

**1 psi = 6.89476 kPa**

# Strength

Strength is a material's ability to resist failure under load.

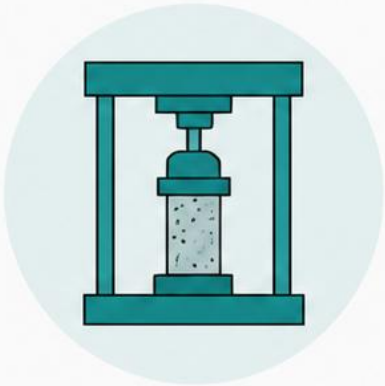
It is expressed as a

**limiting stress**

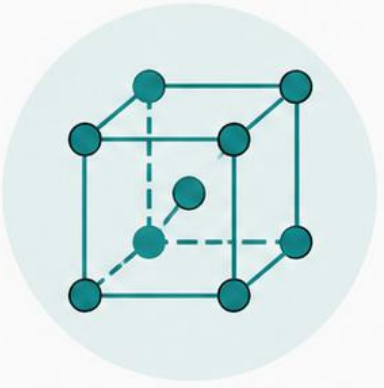
for a specified loading condition.

# Strength

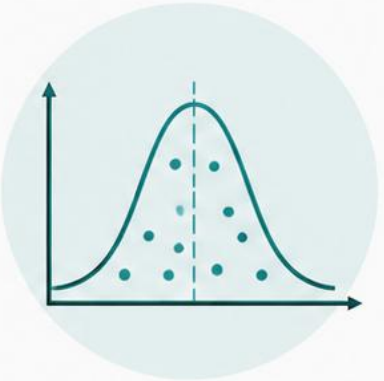
Depending on how strength is determined, we distinguish:



**technical strength**



**theoretical strength**



**statistical strength**

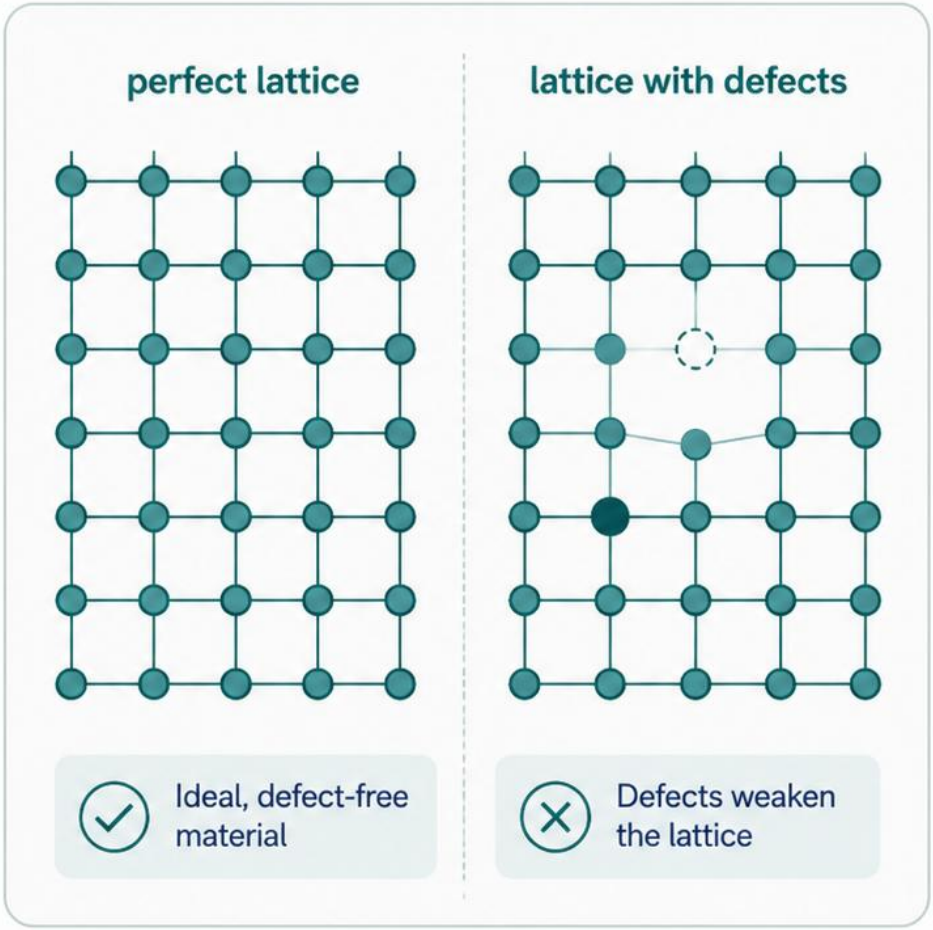
# Theoretical strength

The maximum strength of an ideal, defect-free material

Estimated from the number and strength of interatomic bonds.

Actual strength is much lower because real materials contain defects.

**Defects reduce the actual strength of materials.**



# Technical strength

Strength determined by testing a **real material specimen.**

**Measured experimentally** on a test specimen.

The specimen should be **representative** of the material.

Measured strength depends on **specimen geometry and test conditions.**

**Example test:**

Compression test in a universal testing machine.

↓

The load at failure is used to calculate the strength.

**Common specimen geometries**

Cube

Cylinder

# Test samples

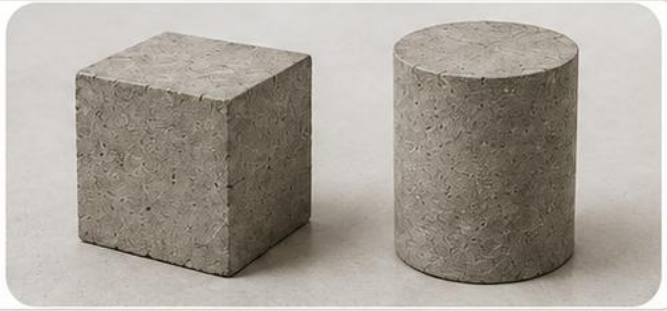
Test specimens can be obtained in several ways:

**1. Shaped from the material**

e.g. cutting, coring, drilling

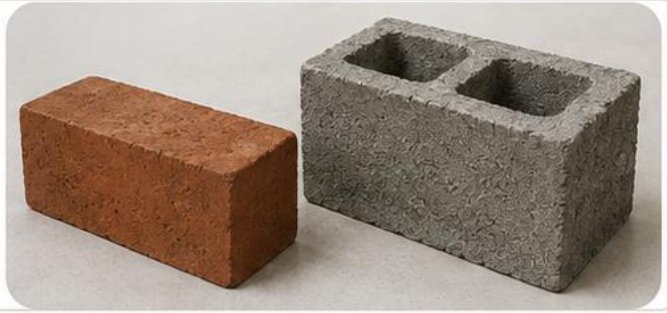
**2. Made directly in the required shape**

e.g. cubes, cylinders



**3. Testing whole products**

e.g. bricks, blocks



# Statistical strength

The properties of a whole population can be estimated from results obtained on a sample of test specimens using **statistical methods**.

**SAMPLE**

Results from a limited number of tests

**STATISTICAL METHODS**

Statistical analysis of the test results

**POPULATION**

Estimate of the properties of the whole population

Statistical methods allow us to **generalize** from a sample to the entire population.

# Statistics

> **The only statistics you can trust are those you falsified yourself.**
>
> – *Winston Churchill*

Statistics turn **data** into **knowledge** and help us **understand** and **improve** reality.

# Statistics - glossary

1

## Random experiment

A process or trial whose outcome cannot be predicted with certainty in advance.

2

## Population

The entire set of items or observations we want to study.

3

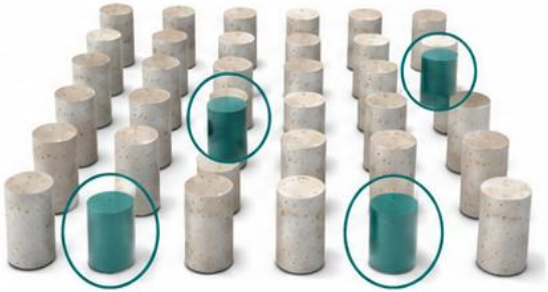
## Simple random sample

A sample selected from a population using a random procedure so that every possible sample of a given size has an equal chance of being chosen.

# Statistical evaluation of strength

**1**

Only part of the population is tested – a **random sample**.

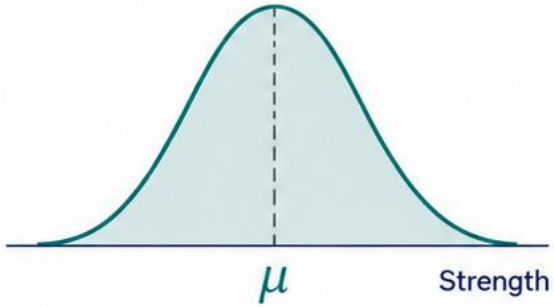


**2**

From the results of a random sample, a corresponding **population parameter** can be estimated.

**3**

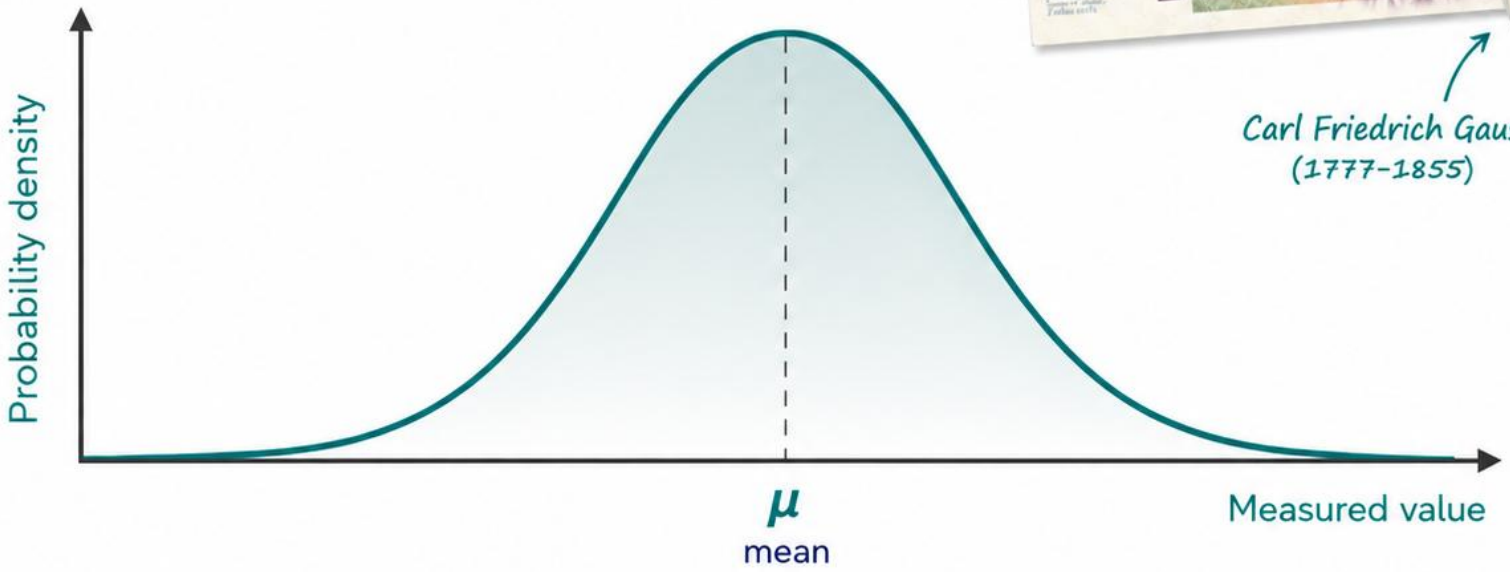
In many practical cases, strength values are approximated by a **normal distribution** (Gaussian distribution).



# Normal distribution

A continuous, symmetric distribution commonly used to describe **variability in measured properties**.

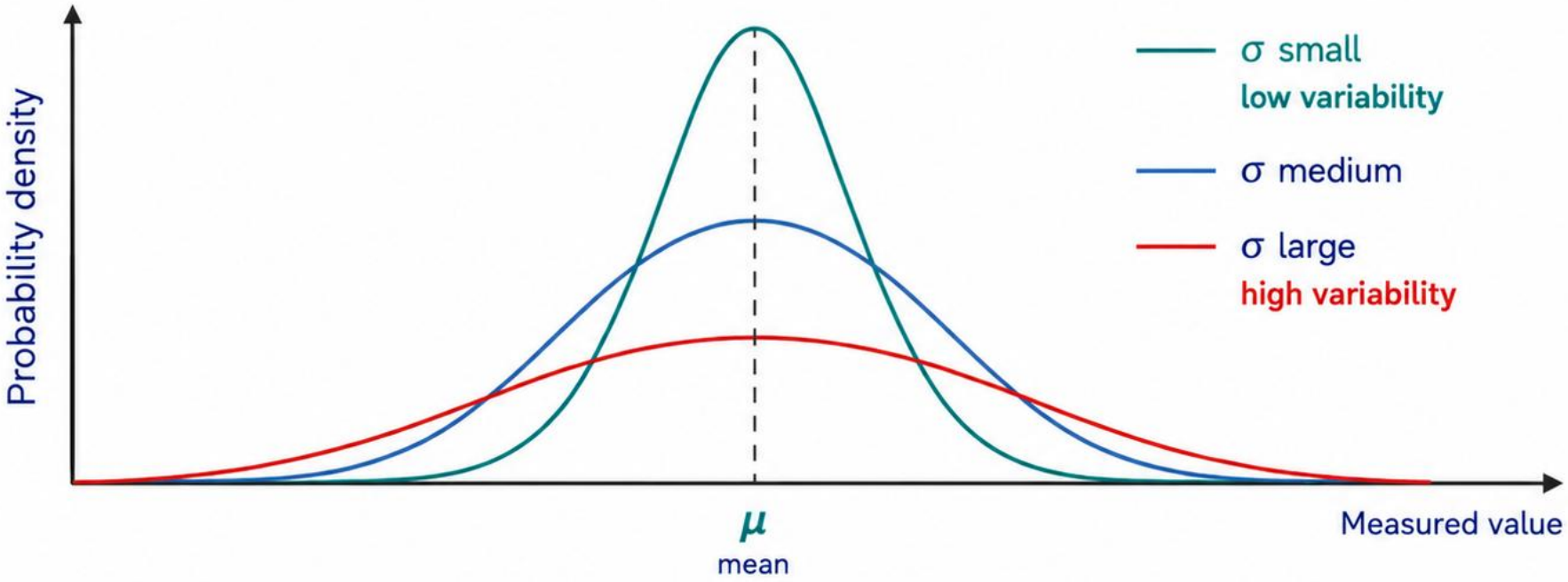
*Carl Friedrich Gauss*
*(1777-1855)*




Symmetric about the mean $\mu$

# Effect of standard deviation (σ)

The standard deviation determines the spread of the normal distribution.
Smaller σ → **less variability**, larger σ → **more variability**.



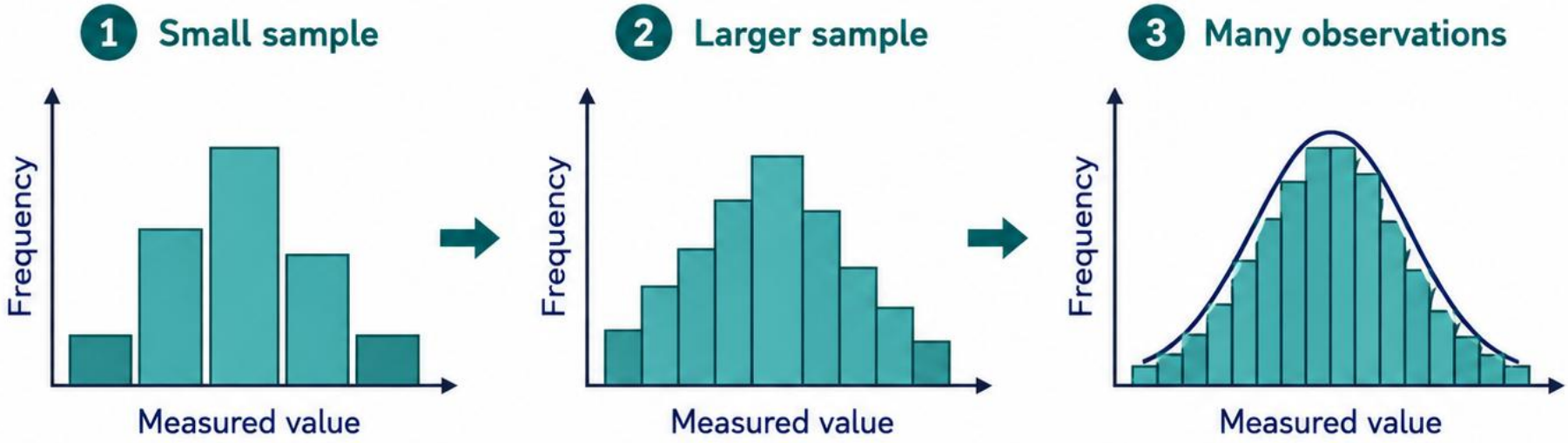

Smaller σ → narrower distribution → **lower variability**

Larger σ → wider distribution → **higher variability**

# Histogram

A histogram shows the distribution of measured values in a sample.

Values are grouped into intervals (bins), and the bar height represents their frequency.

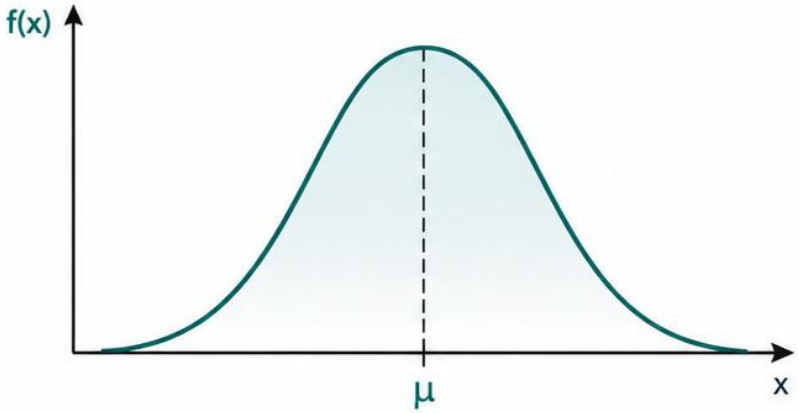


More observations generally provide a more stable picture of the underlying distribution.

# Normal and other distributions

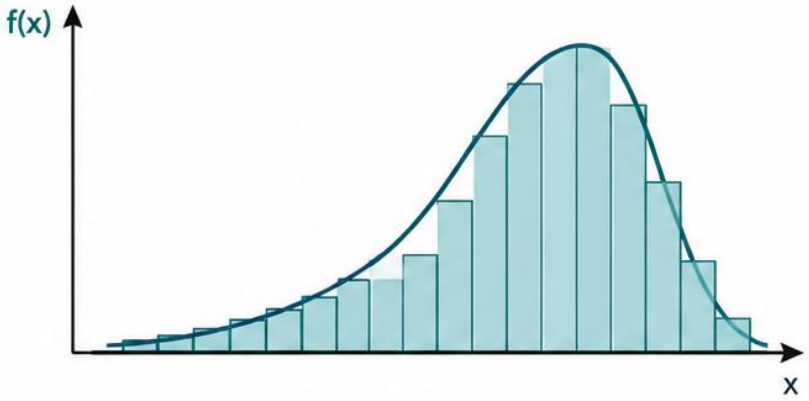
- **Normal distribution — symmetric**

  A symmetric bell-shaped curve.




- **Skewed distribution — asymmetric**

  A non-symmetric distribution (right-skewed).




In a normal distribution:

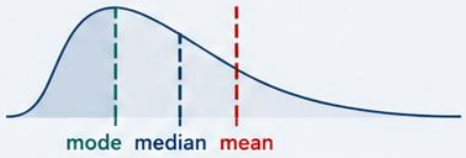
**mean = median = mode**

In a right-skewed distribution:

**mode < median < mean**

The mean is pulled toward the long tail.

Mode is at the peak, median is between mode and mean.

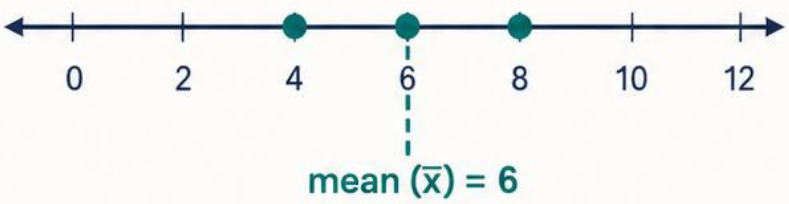


# Same mean – different variability

Two samples can have the same mean, but very different spread.

## Sample A

4 8 6




mean $\bar{x} = 6$

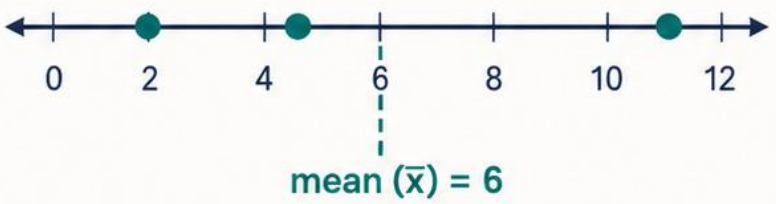
sample variance $s^2 = 4$

standard deviation $s = 2.00$

*smaller spread*

## Sample B

2 5 11




mean $\bar{x} = 6$

sample variance $s^2 = 21$

standard deviation $s = 4.58$

*larger spread*

The mean describes the centre — variance and standard deviation describe the variability.

# Sample statistics

These parameters describe the center and variability of a sample of ***n*** observations.

## Mean

Measures the central value.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\bar{x}$ … sample mean

$x_i$ … $i$-th observation

$n$ … sample size

## Variance

Measures the variability around the mean.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

$s^2$ … sample variance (uses $n-1$ for an unbiased estimate)

## Standard deviation

Typical (RMS) deviation from the mean.

$$s = \sqrt{s^2}$$

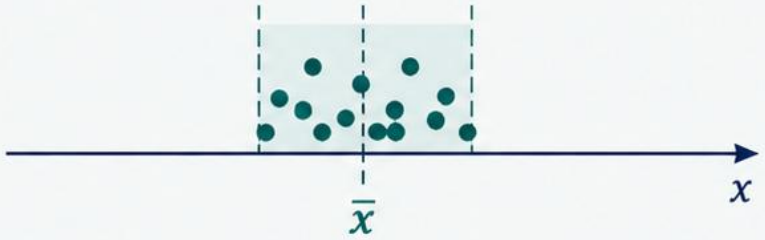
$s$ … sample standard deviation

The standard deviation expresses the **typical spread** of values around the mean.

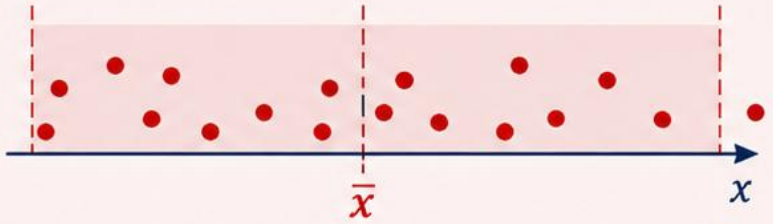
# Standard deviation *s*

Sample standard deviation describes how **far** measured values typically lie **from the mean.**

**Low standard deviation**
values are clustered close to the mean




**High standard deviation**
values are spread over a wider range

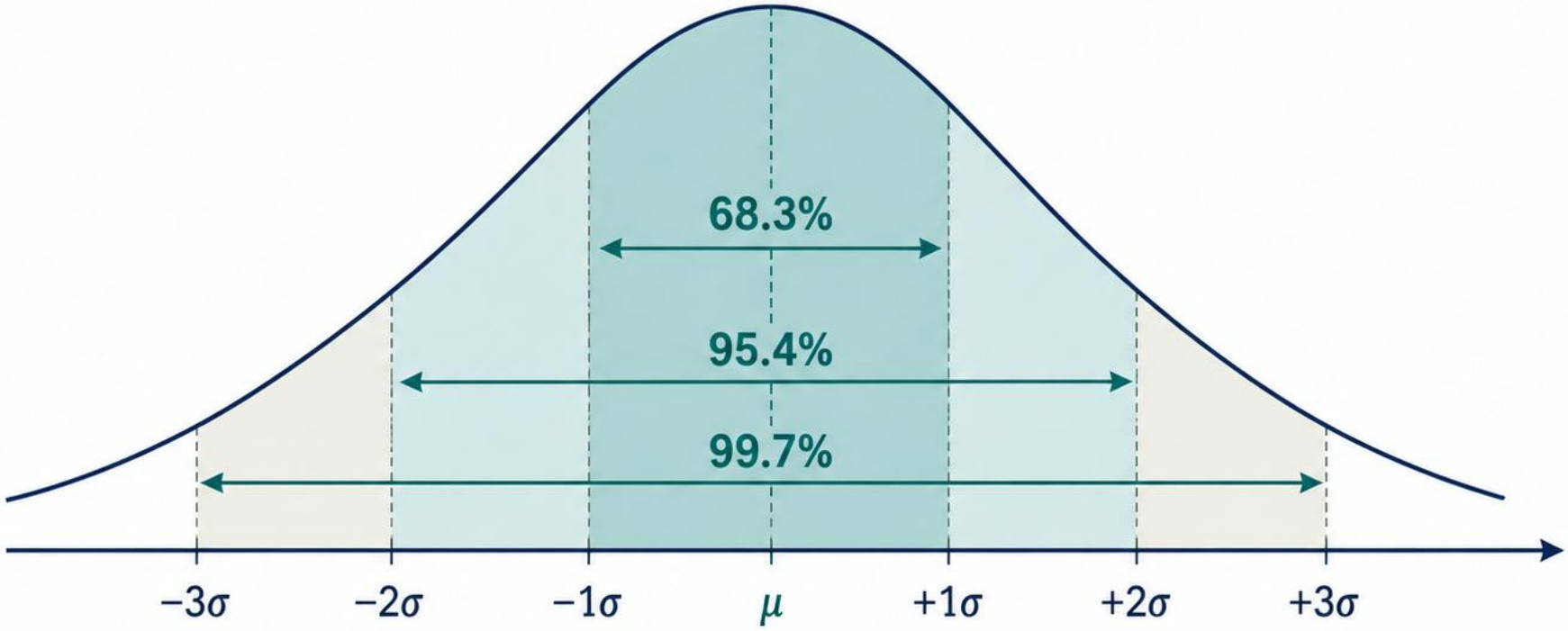



**Smaller $s$** = lower scatter; **larger $s$** = higher scatter.

# Normal distribution

**For a normal distribution:**

- **68.3%** of values lie within $\mu \pm 1\sigma$
- **95.4%** of values lie within $\mu \pm 2\sigma$
- **99.7%** of values lie within $\mu \pm 3\sigma$



# Characteristic value

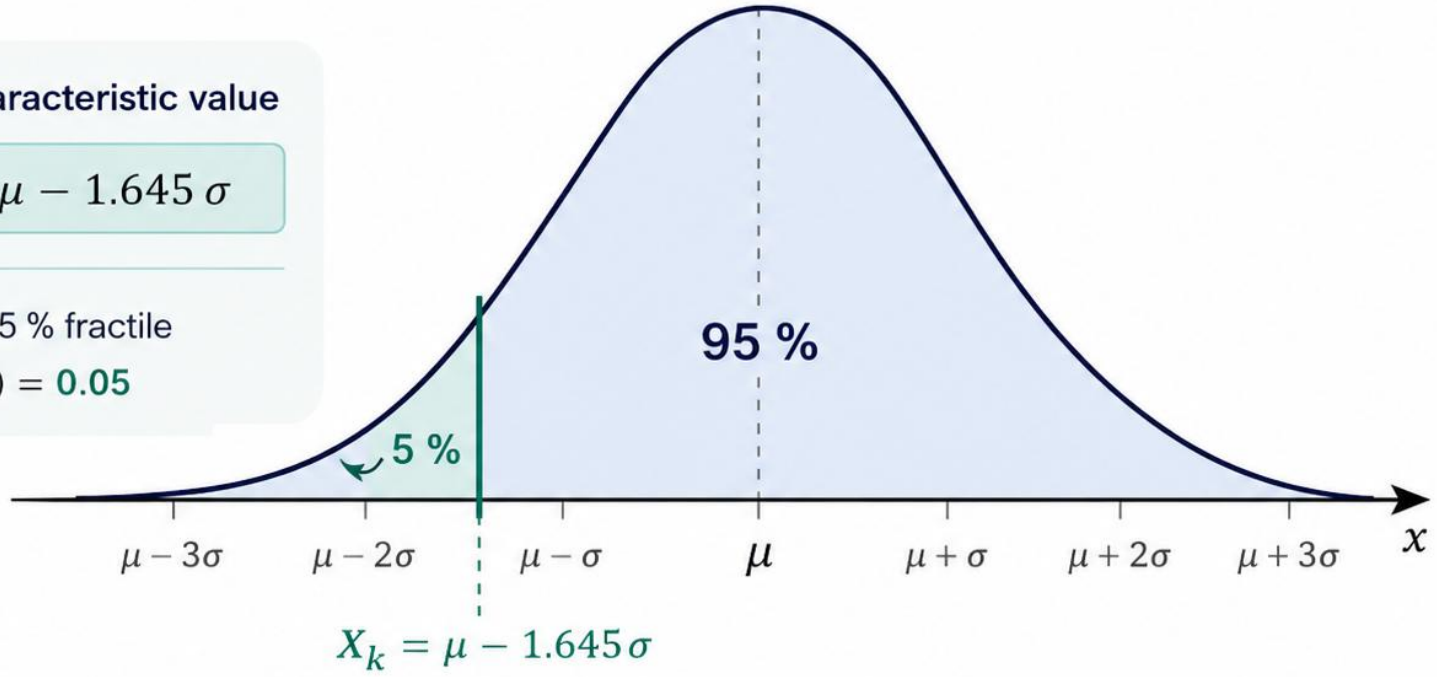
For a (known) normal distribution, the 5 % lower fractile is used as the **lower characteristic value**.

**Lower characteristic value**

$$X_k = \mu - 1.645\,\sigma$$

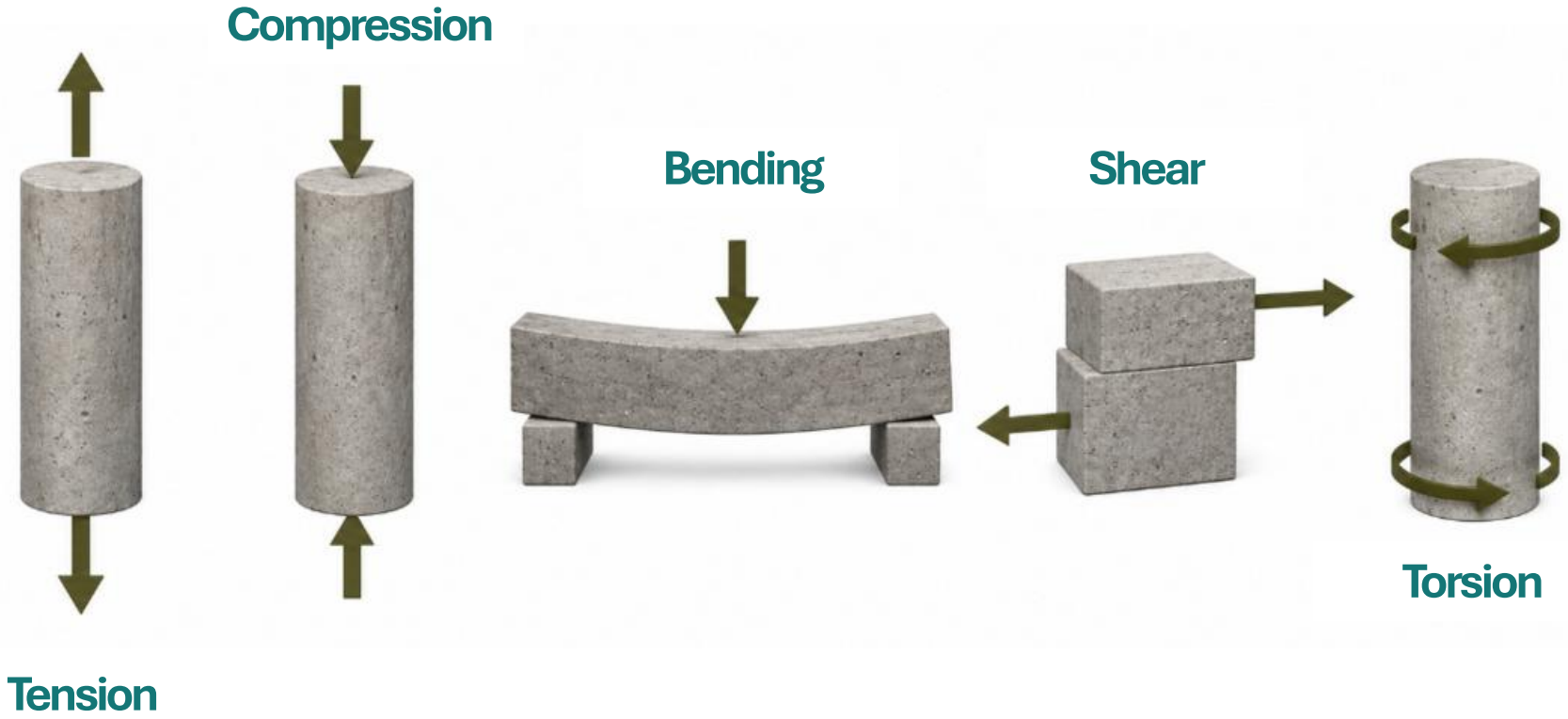
**1.645** ... 5 % fractile

$P(X \leq X_k) = 0.05$




$X_k$ is the value that **95 %** of results are expected to exceed.
It represents a conservative (guaranteed) lower bound.

# Strength testing by loading mode



# Compressive strength

> Maximum compressive stress sustained by a specimen during a compression test.

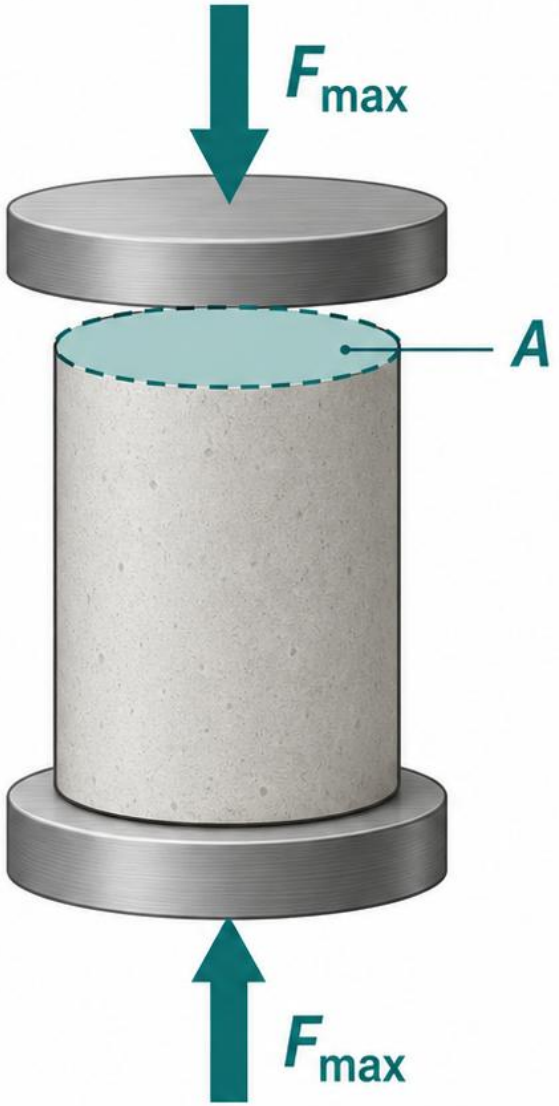
$$f_c = \frac{F_{max}}{A}$$

$F_{max}$ — maximum load at failure [N]

$A$ — loaded cross-sectional area [mm²]

$f_c$ — compressive strength [MPa]

ⓘ 1 MPa = 1 N/mm²



# Compressive strength - testing

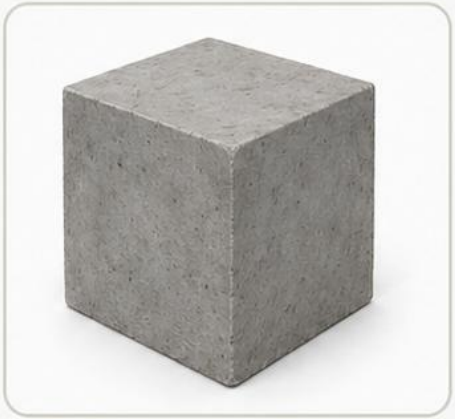
A hydraulic pump or electric motor applies a steady, controlled pressing force until the sample fractures.

# Test specimens for compression testing

## Common specimen types

- **Cubes** — concrete
- **Cylinders** — concrete, lightweight concrete
- **Prisms / beams** — lightweight concrete
- **Prism halves** — cement mortar
- **Whole products** — blocks, bricks

*Typical shapes used in practice*



# Compression testing of prism halves

After the flexural test, the broken prism halves can be used for compression testing.

- Load is applied through **40 × 40 mm auxiliary plates.**

$$A = 40 \times 40 = 1600 \text{ mm}^2$$
(loaded area)

The loaded area is defined by the **auxiliary plates**, not by the entire specimen surface.

# Concrete compressive strength classes

## C25/30

Ø150 × 300 mm

**Cylinder**

**25 MPa**

characteristic cylinder strength

$h/d = 2$

<

cylinder strength < cube strength

150 × 150 × 150 mm

**Cube**

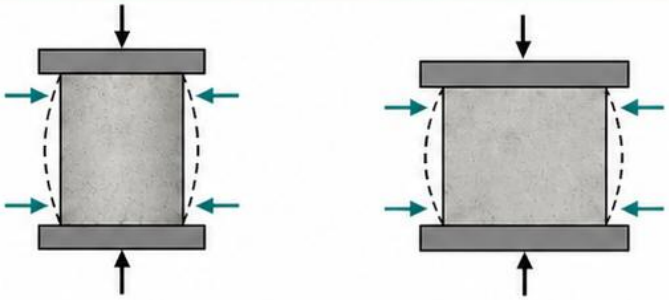
**30 MPa**

characteristic cube strength

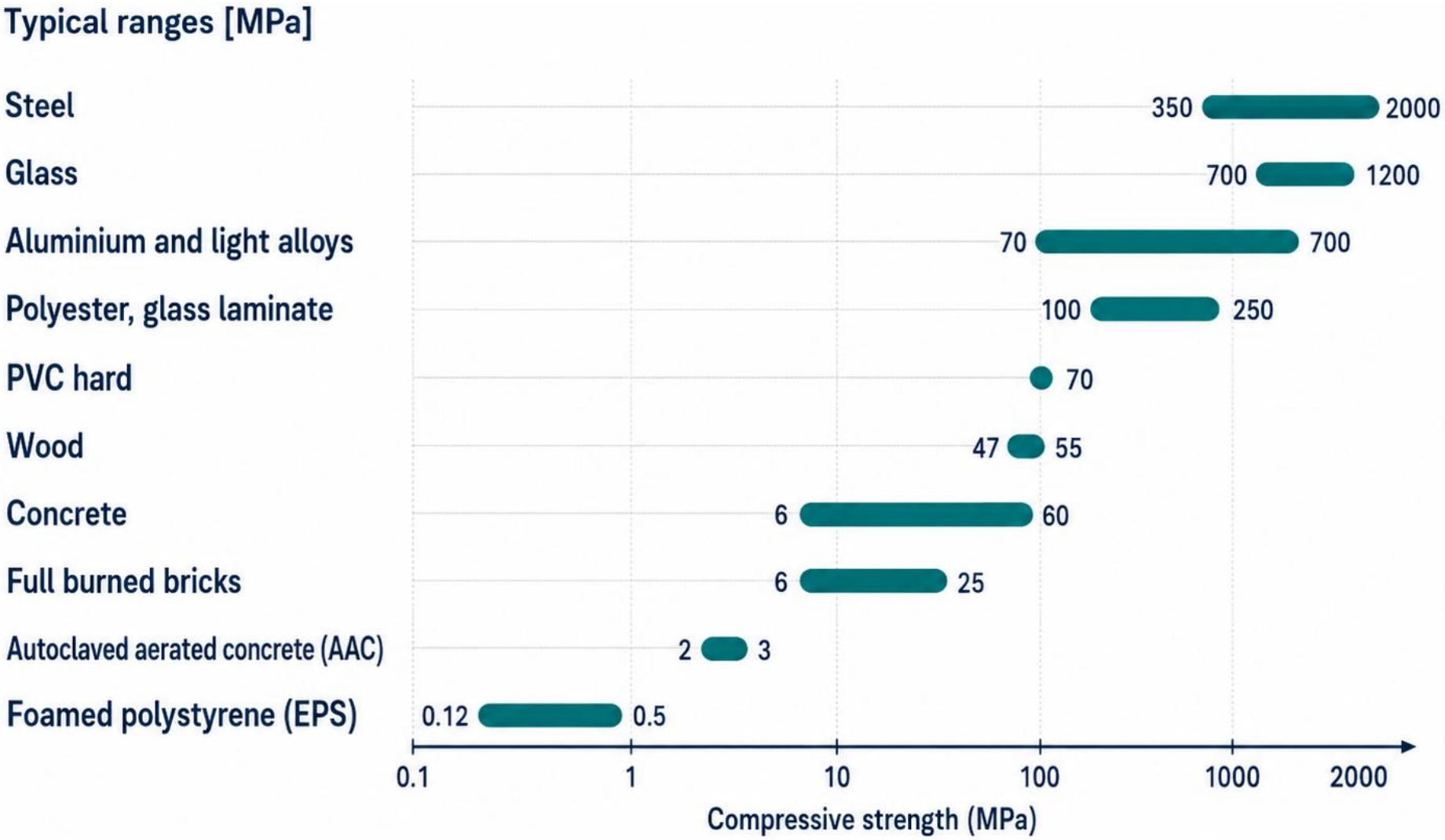
$h/b = 1$

*standard test specimens*

The difference is mainly caused by specimen **geometry** and end restraint due to **friction** at the loading platens.



# Compressive strength of building materials




*Values taken from course cheat sheet*

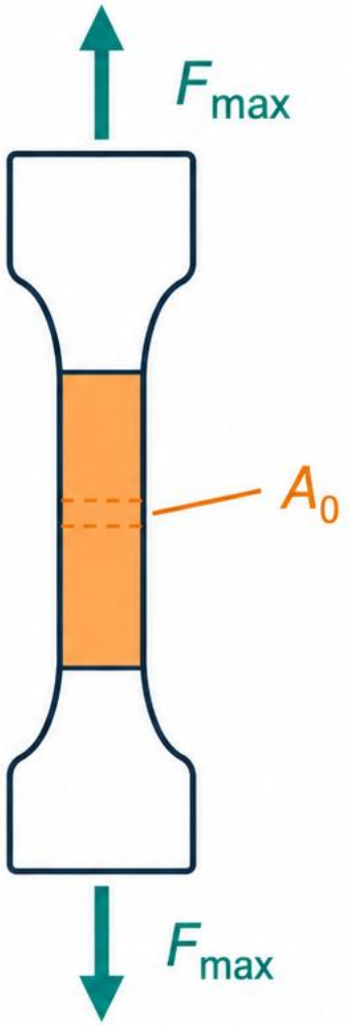
# Tensile strength

maximum resistance of a material to axial tensile loading

$$R_t = \frac{F_{max}}{A_0} \quad [\text{MPa}]$$

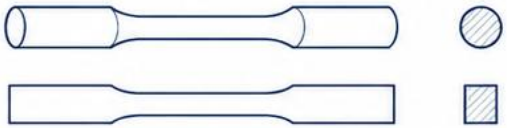
$F_{max}$ … maximum force [N]

$A_0$ … original cross-sectional area [mm²]



# Tensile test specimens

- **round or flat specimens**

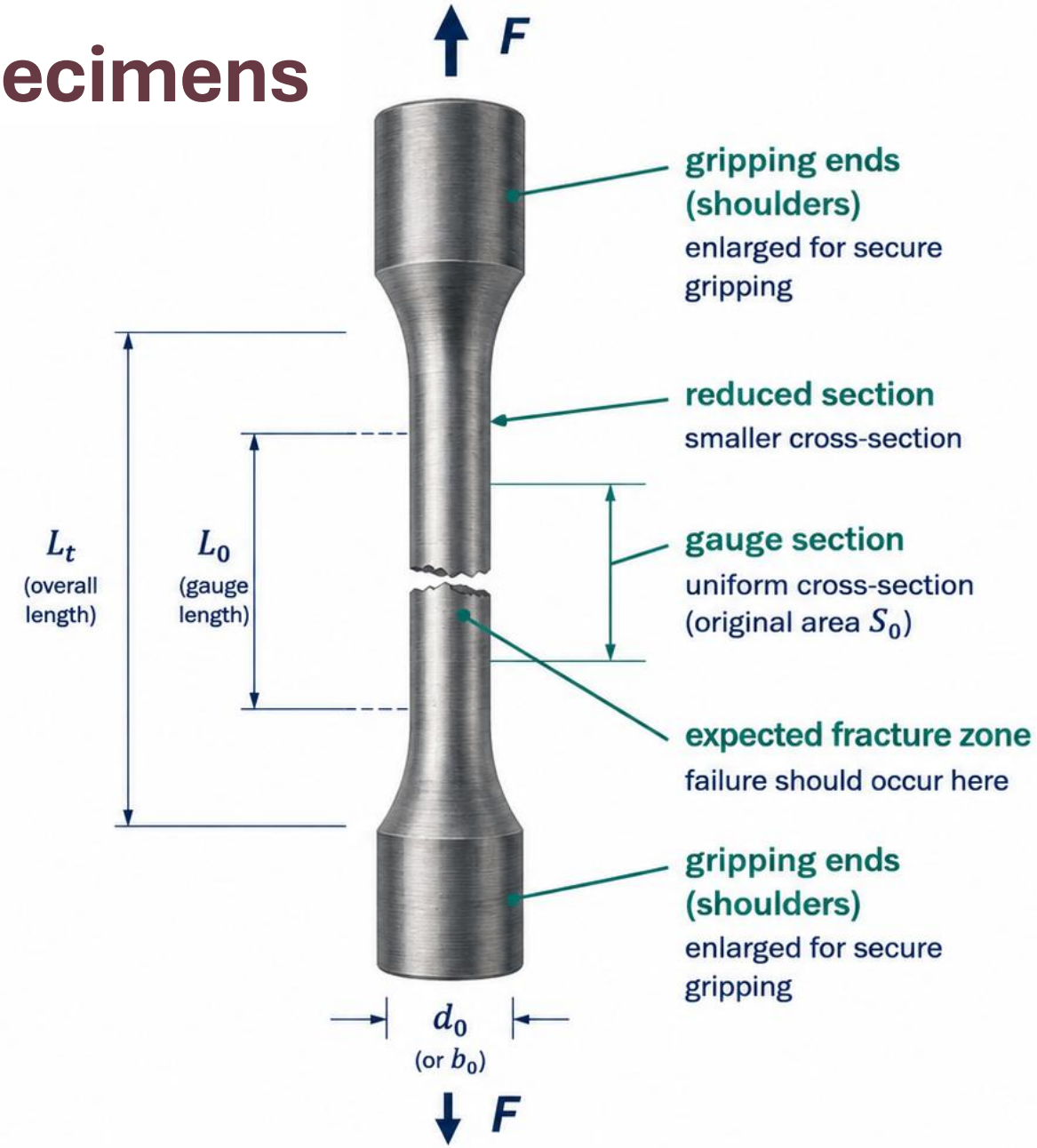


- **reduced central section**
  ensures failure in the gauge section
- **enlarged gripping ends / shoulders**
  provide secure gripping in the testing machine
- **failure should occur in the gauge section**

**Tensile strength**

$$R_m = \frac{F_m}{S_0} \quad \text{MPa}$$

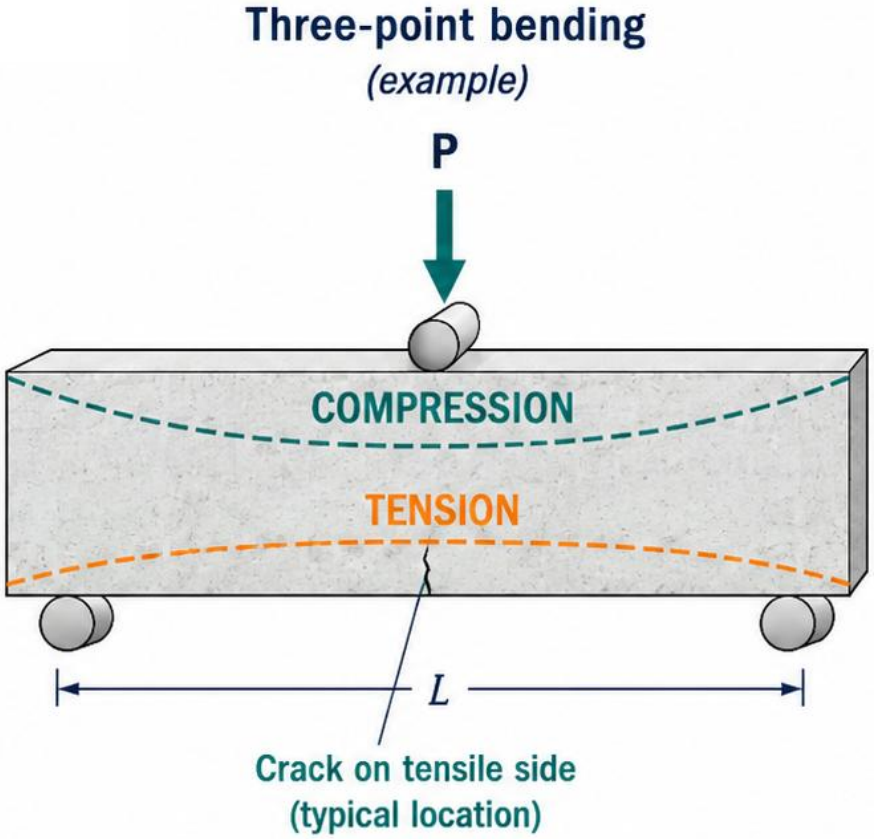
$F_m$ ... maximum force [N]
$S_0$ ... original cross-sectional area [mm²]



# Flexural strength

Maximum **flexural stress** sustained by a specimen during a bending test.

- Bending produces **compression** on one side and **tension** on the opposite side.
- For brittle materials, failure usually initiates on the **tensile side**.




Flexural strength is determined from the maximum load at failure and the specimen geometry (per relevant standard, e.g. **EN 12390-5**).

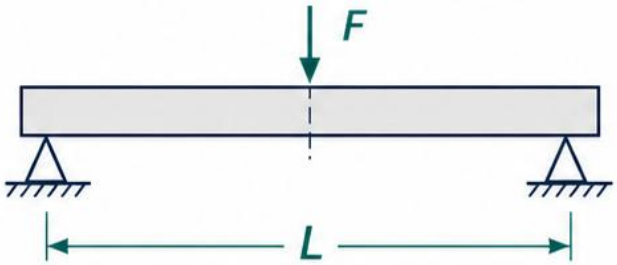
# Flexural strength

$$f_f = \frac{M_{max}}{W}$$

$M_{max}$ — **maximum bending moment [N·mm]**
depends on the loading arrangement

$W$ — **section modulus [mm³]**
depends on the shape and dimensions of the cross-section

**1) Three-point bending**




$$M_{max} = \frac{FL}{4}$$

**2) Symmetric four-point bending**

$$M_{max} = Px$$

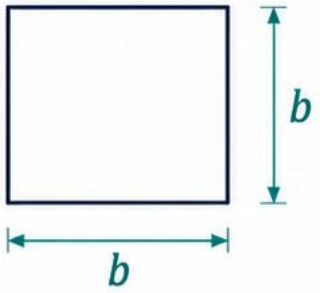
$P$ = load at each loading point

Flexural strength is determined by both the loading arrangement and the cross-section geometry.

# Section modulus W

Section modulus depends only on the shape and dimensions of the cross-section.

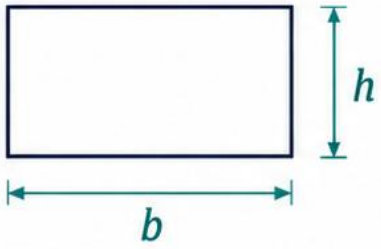
## 1. Square section




$$W = \frac{b^3}{6}$$

[mm³]

## 2. Rectangular section

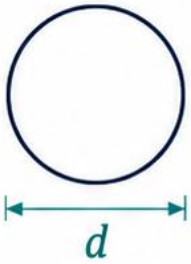



$$W = \frac{bh^2}{6}$$

$h$ = bending direction

[mm³]

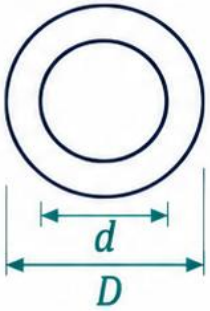
## 3. Solid circular section




$$W = \frac{\pi d^3}{32}$$

[mm³]

## 4. Hollow circular section




$$W = \frac{\pi (D^4 - d^4)}{32D}$$

[mm³]

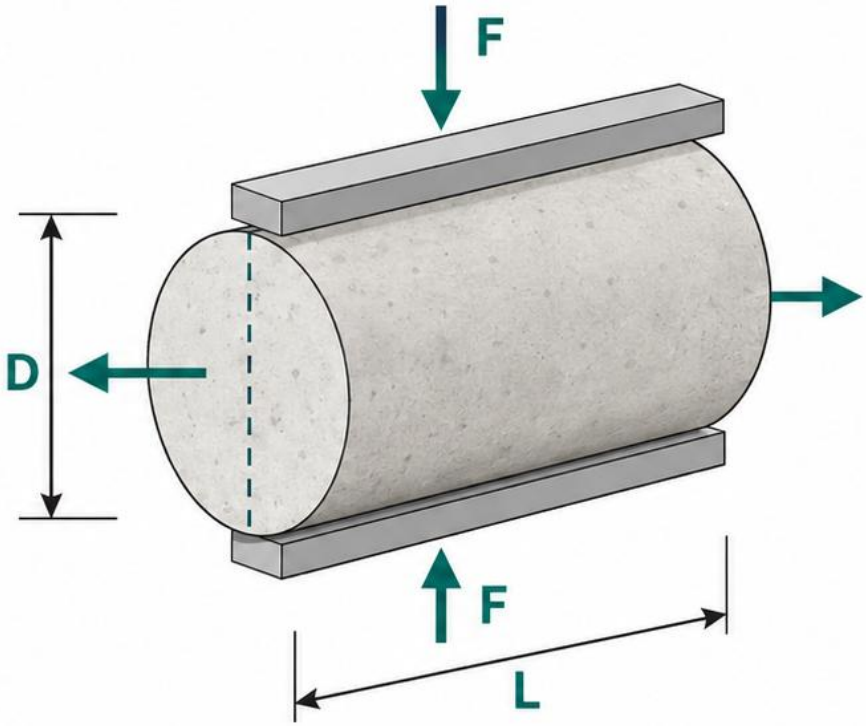
For the same load, a larger $W$ reduces flexural stress.

# Splitting tensile strength

- indirect tensile test for brittle materials
- compressive line load induces tensile stress across the diameter
- failure occurs by splitting along the loaded diameter

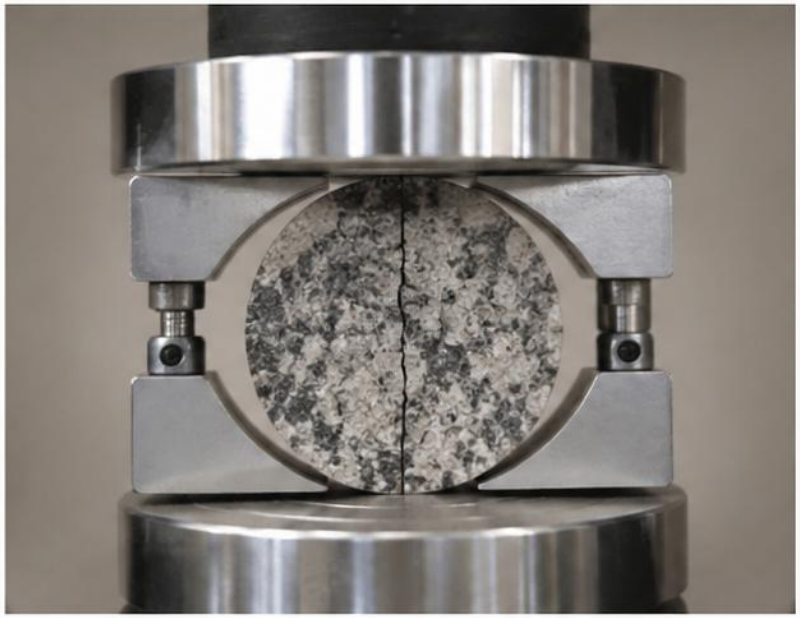
$$R_t = \frac{2F_{max}}{\pi \cdot D \cdot L} \quad [\text{MPa}]$$

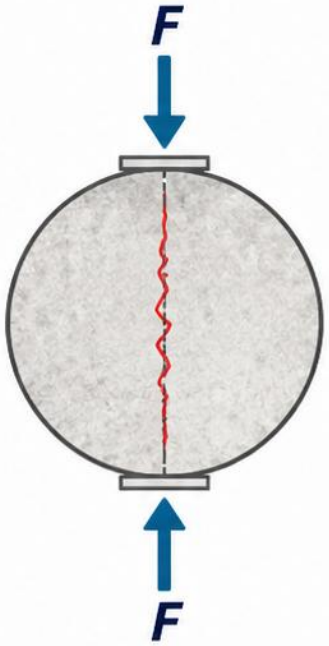
| | |
|---|---|
| $F_{max}$ | maximum load [N] |
| $D$ | specimen diameter [mm] |
| $L$ | specimen length [mm] |



# Brazillian test for rocks

A widely used indirect method for determining the tensile strength of brittle rock.






Compressive load applied along the diameter.

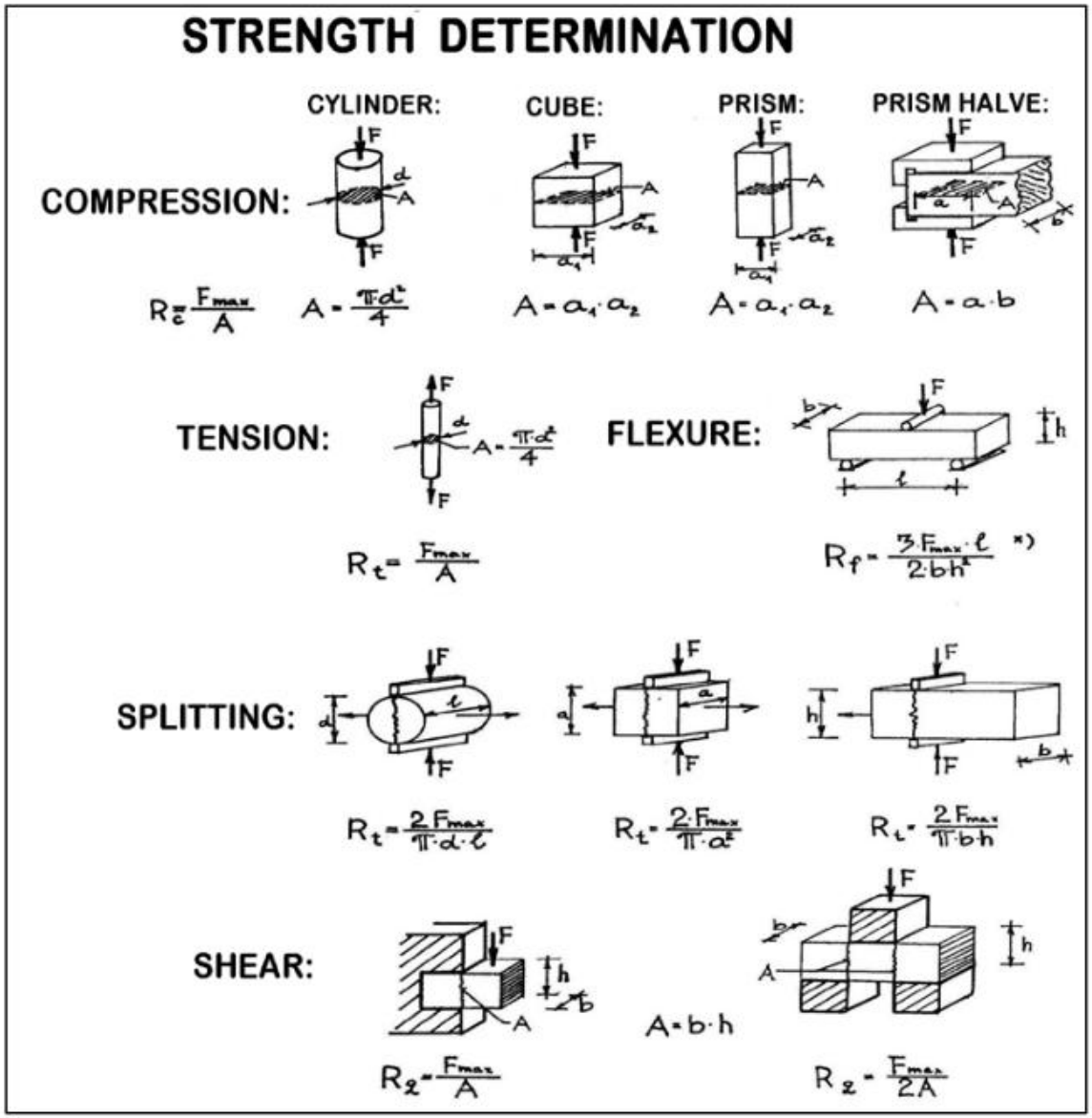
- Tensile stress develops perpendicular to the load.
- Fracture occurs along the loaded diameter.

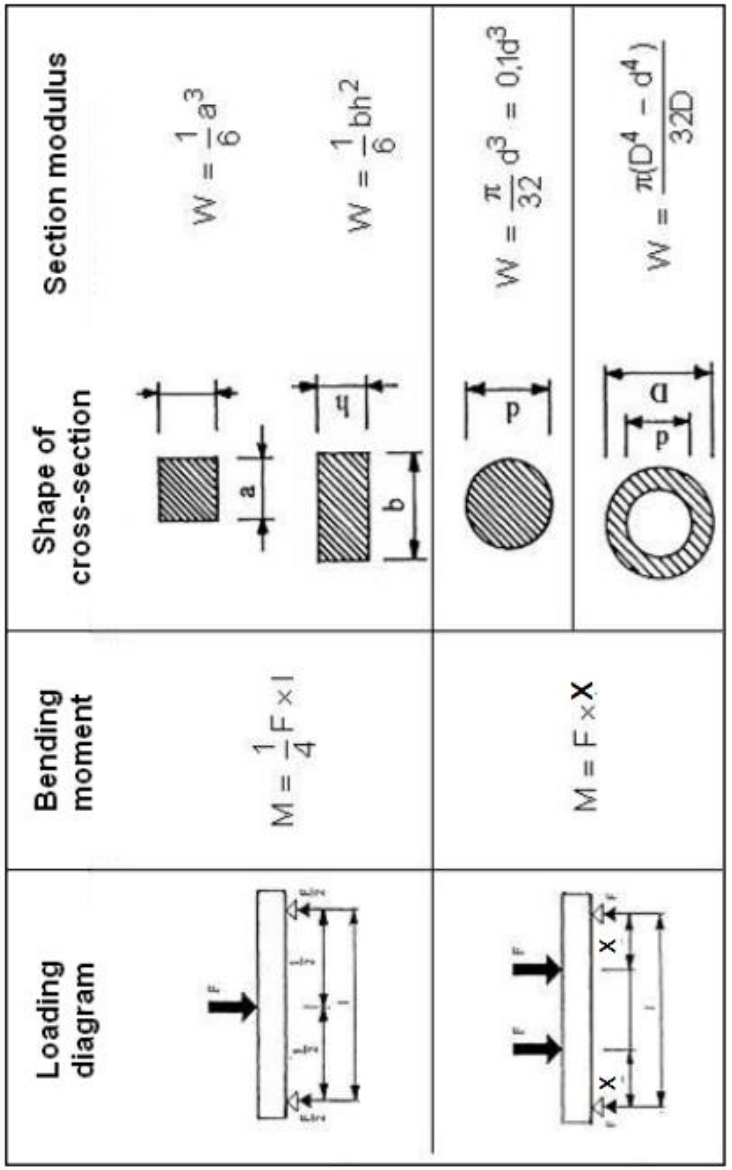
**Compression**
High resistance
(tens to hundreds of MPa)

**Tension**
Much lower resistance
(typically 1–10 MPa)

ASTM D3967-23 – Splitting tensile strength of intact rock core specimens

# Test aid


STRENGTH DETERMINATION
CYLINDER:
CUBE:
PRISM:
PRISM HALVE:
COMPRESSION:
TENSION:
FLEXURE:
SPLITTING:
SHEAR:


Section modulus
Shape of cross-section
Bending moment
Loading diagram